

Jets from Collapsing Bubbles

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Abstract

When an asymmetric bubble collapses it generally produces a well defined high velocity jet. This is remarkable because one might expect such a collapse to produce a complex or chaotic flow rather than an ordered one. I present a dimensional argument for the ubiquity of jets from collapsing bubbles, and model the aspherical collapse of a bubble with pieces of Rayleigh's solution for spherical collapse and its cylindrical analogue. This model explains the ubiquity of jet formation in aspherical collapse, and predicts the shape and velocity profile of the resulting jet. These predictions may be tested in the laboratory or by numerical calculation. An application to solid spall is suggested.

I. Introduction

The aspherical collapse of a bubble or void in a liquid produces a fast liquid jet^{1–3}. This phenomenon is remarkably robust. It occurs for axially symmetric collapse of a single bubble near a solid wall or free surface. It occurs in at least some of the bubbles produced in turbulent cavitating flow, but it is apparently unknown whether it occurs in all such bubbles, or what initial conditions are required.

Jet production is of great technological importance. Jets are the means by which cavitation damages nearby solid surfaces^{2,4}. Fast jets are deliberately produced by shaped charges⁵, and are remarkably insensitive to their geometry. Jets launch droplets from the sea surface⁶, producing marine salt aerosols. Jets are also responsible for the sensitization of explosives by microscopic bubbles^{7–8}.

There are a number of elegant analytic theories of jet production^{9–11}. Numerical calculations of axisymmetric aspherical collapse^{12–15} readily show jet formation. However successful these theories and calculations, they do not explain the robustness of the phenomenon and the applicability of these somewhat idealized results to the bubbles encountered in practice: jets form from the collapse of bubbles which cannot be expected to be symmetric, and despite the best efforts of engineers to prevent them. A qualitative model of jet formation might help understand why it is so ubiquitous.

A simple analytic solution is possible for the collapse of a spheroidal bubble. Laplace's equation for the velocity potential ψ separates in spheroidal coordinates¹⁶ (either prolate or oblate) (u, v, θ) . A spheroidal bubble is characterized by $\psi = \psi(u)$. It is readily seen by explicit differentiation that its aspect ratio does not change during its collapse, so that if it remains spheroidal it will not produce a jet. More generally, no bubble which possesses inversion symmetry will produce a jet, because any jet would stagnate against its mirror image jet upon convergence. In fact, inversion symmetry may be broken by the presence of a nearby wall or free surface, or by the growth of small perturbations, and initially spherical or ellipsoidal bubbles do produce jets, as is seen in the numerical calculations.

II. Why Jets

A dimensional argument can be made for the ubiquity of jet formation. Suppose the collapsing bubble is initially approximately spherical, so that at each point on its surface the two radii of curvature are comparable to each other and have roughly the same magnitude everywhere on the surface. Then only one quantity with the dimensions of length (the approximate initial radius) is defined. The only other independent dimensional quantity is a velocity $c \equiv (P/\rho)^{1/2}$, where ρ is the liquid density and P the pressure at infinity. If viscosity and surface tension are neglected and the bubble contains no uncondensable gas there are also no characteristic dimensionless numbers. For some geometries (spheres and spheroids, for example) void collapse will be self-similar, maintaining the shape of the bubble.

At a specified elapsed time t a new length scale ct is defined which is characteristic of that time, but not of the collapse process as a whole. If the bubble shape is to undergo a qualitative change (such as the formation of a jet) its description would require at least one additional characteristic length $r'(t)$, typically a radius of curvature. If $r'(t)$ is time-dependent it may be constructed from ct . However, it is not possible to define an additional constant r'_0 which is characteristic of the process as a whole (rather than a specific time), because the initial conditions do not contain enough information; a limiting, final or time-independent radius of curvature would be an example of such a forbidden parameter.

A spherical vacuum bubble satisfies this condition by collapsing to a point, rather than reversing its collapse at a finite radius r'_0 . If a collapsing void has an asymmetry or a dimple or pimple on its wall and does not preserve its proportions, the asymmetry must either decay or sharpen until the flow becomes singular and a cusp forms. For this reason a growing asymmetry will generally lead to a jet which develops singular conditions at its tip. This argument for jet formation also applies to bubbles near walls or other bubbles (usually the source of asymmetry), if all the initial characteristic lengths are comparable to the collapsing bubble's initial radius.

III. Model

I suggest the following model of jet production: If the two principal radii of curvature of a bubble are nearly equal, aspherical collapse is locally approximated by spherical collapse, as described by a modified version¹⁷ of Rayleigh's classic theory¹⁸, while if the principal radii of curvature are very different it is locally approximated by cylindrical collapse. Collapse of a finite angular range of a cylinder produces a sheet jet (as in a linear shaped charge⁵) rather than the linear jet produced by axially symmetric collapse of an entire cylinder¹³.

In this elementary model the difference between spherical (or cylindrical) collapse and that of an aspherical bubble is that in the aspherical case different portions of the surface converge to the center at different times. Instead of meeting an oppositely directed convergent flow from the other side, and stagnating against it in a central pressure peak (as happens in inversion-symmetric collapse), in the asymmetric case the fluid which converges first forms a fluid jet which then penetrates the unconverged fluid approaching from the opposite side. Jets are likely to be produced by the collapse of any bubble without inversion symmetry.

This model is applicable not only to bubbles and to voids in explosives, but also to hemispherical shaped charges. It is not applicable to conical shaped charges, which are not locally spherical, and whose convergence is not even locally cylindrical at their apices; a cone defines no quantities with the dimensions of length, and contains a geometric singularity in its initial state.

In the frequently encountered case of a bubble near a plane solid boundary or free surface the collapse is azimuthally symmetric about the surface normal, and the spherical solution is applicable to the fastest-collapsing portion of the bubble. The Rayleigh solution for the velocity field surrounding a spherical void, which has collapsed from an initial radius

R_0 to a radius R , is

$$v(r) = \left[\frac{2}{3} \frac{P}{\rho} (R_0^3 - R^3) R \right]^{1/2} r^{-2} \equiv \frac{C_3}{r^2} \quad (r \geq R), \quad (1)$$

where here P is the difference between the pressure at infinity and the pressure in the void. In the limit $R/R_0 \rightarrow 0$, $C_3 \rightarrow [2PR_0^3R/(3\rho)]^{1/2}$. The distribution of mass in a spherical cap of solid angle Ω with respect to specific kinetic energy $\mathcal{E} \equiv \rho v^2/2$ is then

$$\frac{dM}{d\mathcal{E}} = \frac{\Omega}{2^{11/4}} \frac{\rho^{7/4} C_3^{3/2}}{\mathcal{E}^{7/4}} \quad \left(\mathcal{E} \leq \frac{\rho C_3^2}{2R^4} \right). \quad (2)$$

Performing the integral $\int \mathcal{E}(M) dM \sim \mathcal{E}^{1/4}$ demonstrates that the kinetic energy is weakly concentrated in the fluid with the greatest specific kinetic energy; that is, at the tip of the jet.

During collapse of a cap spherical convergence is assumed (otherwise (1) and (2) would not be applicable), but after convergence this can no longer be the case. I assume that the fluid then forms a parallel jet, with the distribution of speed and kinetic energy given by (1) and (2). This is not required by any conservation law, even for a perfect fluid, but is the simplest possible assumption. It is plausible for a cap of small Ω because the convergent velocities are nearly parallel and are readily collimated, and because in a narrow jet the zero pressure boundary condition along its sides ensures that any longitudinal pressure gradient and acceleration are small.

It is necessary to introduce an upper cutoff \mathcal{E}_{max} on \mathcal{E} (or, equivalently, a lower cutoff $R_{min} = (P/3\mathcal{E}_{max})^{1/3} R_0$ on R), because otherwise all the kinetic energy would appear in an infinitesimal mass of fluid. This cutoff may be the consequence of the onset of compressibility (surface tension and viscosity are readily verified to be negligible in the converged flow if they were negligible in the original bubble) or a breakdown in the geometric assumptions. The resulting value

$$C_3 = \left(\frac{2}{\rho} \right)^{1/2} \left(\frac{PR_0^3}{3} \right)^{2/3} \mathcal{E}_{max}^{-1/6} \quad (3)$$

is fortunately only weakly dependent on \mathcal{E}_{max} ; the limiting speed $v_{max} \equiv (2\mathcal{E}_{max}/\rho)^{1/2}$. For water, plausible values (assuming compressibility is the limiting mechanism) are $\mathcal{E}_{max} \sim 2 \times 10^{10}$ erg/cm³ and $v_{max} \sim 2 \times 10^5$ cm/sec; $R_{min} \sim 0.026R_0$ if $P = 1$ bar. The high velocity tip of the jet may be difficult to observe, because it is eroded by residual gas in the bubble and by more slowly converging fluid on the opposite side of the bubble.

As the jet propagates it stretches. If its convergence occurs instantaneously and at one point then its radius s at a distance ℓ from that point at a time t after convergence is

$$s = \left(\frac{\Omega}{2\pi} \right)^{1/2} \frac{C_3^{3/4} t^{3/4}}{\ell^{5/4}} \quad (\ell < v_{max}t), \quad (4)$$

which is obtained by changing variables in (2) from \mathcal{E} to v and using $\ell = vt$; the jet terminates at $\ell \sim v_{max}t$. This form is easier to test against laboratory data than (2) because it is easier to measure the shape of a bounding surface than a fluid velocity.

In the case of cylindrical symmetry the solution analogous to (1) for the velocity field is

$$v(r) = \left[\frac{P (R_0^2 - R^2)}{\rho \ln(R_\infty/R)} \right]^{1/2} r^{-1} \equiv \frac{C_2}{r} \quad (r \geq R), \quad (5)$$

where R_∞ is an upper cutoff (set by the system size) on the range of the velocity field. In the limit $R/R_0 \rightarrow 0$, $C_2 \rightarrow \{PR_0^2/[\rho \ln(R_\infty/R)]\}^{1/2}$. The distribution of mass with respect to \mathcal{E} is

$$\frac{dM}{d\mathcal{E}} = \frac{\theta \rho^2 C_2^2}{4 \mathcal{E}^2} \quad \left(\mathcal{E} \leq \frac{\rho C_2^2}{2R^2} \right), \quad (6)$$

where θ is the arc of the collapsing portion of a cylinder. The integral $\int \mathcal{E}(M) dM \sim \ln \mathcal{E}$, so that kinetic energy is evenly distributed per decade across the specific energy spectrum. An upper cutoff \mathcal{E}_{max} and a lower cutoff R_{min} are again required as $R \rightarrow 0$. The thickness h of a collapsed sheet is found, in analogy to (4),

$$h = \theta \frac{C_2^2 t^2}{\ell^3} \quad (\ell < v_{max}t). \quad (7)$$

Collapsing bubbles whose rate of convergence is intermediate between cylindrical and spherical in their region of fastest collapse may perhaps be parametrized by solutions of

non-integer dimension n . The velocity field is

$$v(r) = \left[\frac{2(n-2)}{n} \frac{P}{\rho} (R_0^n - R^n) R^{n-2} \right]^{1/2} r^{1-n} \equiv \frac{C_n}{r^{n-1}} \quad (r \geq R). \quad (8)$$

The resulting mass distribution in the limit $R/R_0 \rightarrow 0$ is

$$\frac{dM}{d\mathcal{E}} \propto \left(\frac{\rho}{\mathcal{E}} \right)^{(3n-2)/(2n-2)} C_n^{n/(n-1)} \left(\mathcal{E} \leq \frac{\rho C_n^2}{2R^{2(n-1)}} \right), \quad (9)$$

where the constant of proportionality includes the contributing fraction of the n -sphere. The shape of the jet's cross-section depends on the details of convergence, but with the previous assumptions its cross-sectional area A is

$$A \propto \frac{t^{n/(n-1)}}{\ell^{(2n-1)/(n-1)}} \quad (\ell < v_{max} t). \quad (10)$$

This may be fitted to empirical data or to numerical calculation to determine an effective dimension n of the convergent flow.

IV. Discussion

The models of jets discussed in this paper can be tested by comparison to computed jets and to experiment. The most general form is (10), which introduces the non-integer dimensionality parameter n , but which reduces to the spherical results (1)–(4) for $n = 3$ and to the cylindrical results (5)–(7) for $n = 2$.

A related problem is the production of microscopic particulate spall upon shock reflection from a solid surface, at tensile loads insufficient to disrupt the bulk. This is related to fluid jet formation, because both processes involve concentration of energy. Solid spall is a more complicated phenomenon because it involves materials with finite strength, a variety of heterogeneities in the bulk and at the surface, and (usually) anisotropy. It is unclear whether spall is produced by elastic stress concentration at corners (surface scratches, cracks, grain boundaries, *etc.*), followed by brittle fracture, or by plastic flow convergence and jetting at surface scratches and cracks. The latter process would resemble jet formation upon the collapse of a bubble, with the curved solid surface taking the place of the

bubble surface. The plastic flow and brittle fracture hypotheses may be distinguished by microscopic examination of the surfaces of spall fragments. It might also be informative to do experiments on spall from shocked liquid surfaces and amorphous substances, which may be prepared without surface imperfections or heterogeneities in the bulk.

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